Chapter 2
Section 2.3

Warm-up Problem A. For each of the following functions, give the domain and range, and evaluate at $x = 0$.

(a) $f(x) = \sqrt{x}$

D: $x \geq 0$ R: $f(x) \geq 0$ $f(0)=0$

(b) $f(x) = \sqrt{x} + 2$

D: $x \geq 0$ R: $f(x) \geq 2$ $f(0)=2$

(c) $f(x) = \sqrt{x} - 7$

D: $x \geq 0$ R: $f(x) \geq -7$ $f(0)=-7$

(d) $f(x) = \sqrt{x} + 20$

D: $x \geq 0$ R: $f(x) \geq 20$ $f(0)=20$

Warm-up Problem B. For each of the following functions, give the domain and range, and find the x value for which $f(x) = 2$.

(a) $f(x) = \sqrt{x}$

D: $x \geq 0$ $f(4)=2$
R: $f(x) \geq 0$ $x=4$

(b) $f(x) = \sqrt{x-4}$

D: $x-4 \geq 0 \Leftrightarrow x \geq 4$
R: $f(x) \geq 0$ $f(8)=2$
$x=8$

(c) $f(x) = \sqrt{x+6}$

D: $x+6 \geq 0 \Leftrightarrow x \geq -6$
R: $f(x) \geq 0$ $f(-2)=2$
$x=-2$

(d) $f(x) = \sqrt{x-31}$

D: $x-31 \geq 0 \Rightarrow x \geq 31$
$f(x) \geq 0$ $x-31=4 \Leftrightarrow x=35$
$f(35)=2$

Problem 1. Find explicit formulas (in terms of x) for the following functions, and describe in words how the graphs of the following functions differ from $f(x) = \sqrt{x}$. For example, the explicit form of $g(x) = f(x) + 2$ would be $g(x) = \sqrt{x} + 2$.

(a) $g(x) = f(x) + 1$

Shifted up 1

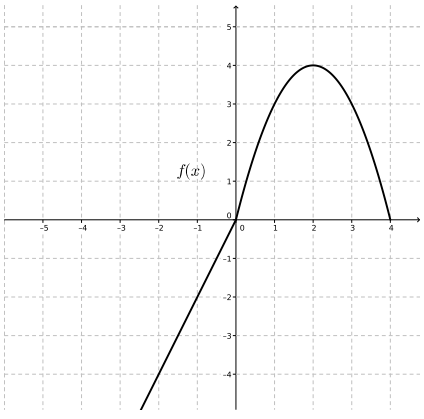
(b) $h(x) = f(x) - 4$

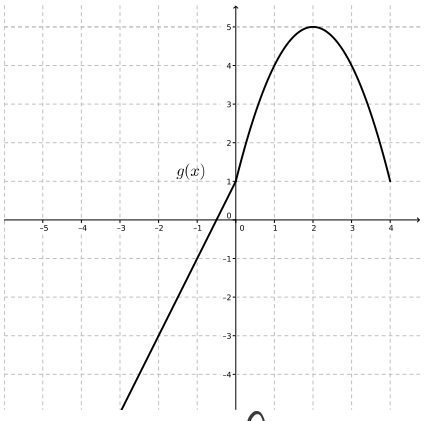
Shifted down 4

(c) $j(x) = f(x) + \sqrt{7}$

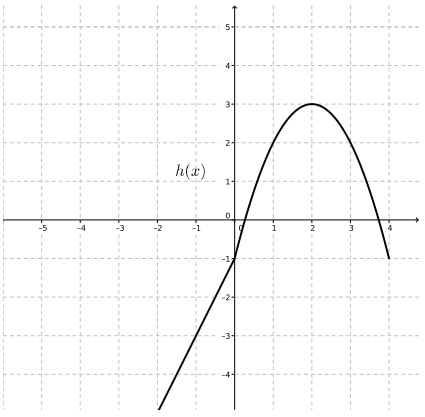
Shifted up $\sqrt{7}$

Problem 2. Consider the graphs of $f(x)$, $g(x)$, and $h(x)$ below. Find formulas for $g(x)$ and $h(x)$ in terms of $f(x)$.





$g(x) = f(x) + 1$



$h(x) = f(x) - 1$

Problem 3. Find explicit formulas (in terms of x) for the following functions, and describe in words how the graphs of the following functions differ from $f(x) = x^2$.

a) $g(x) = f(x+1)$ shifted left 1

$= (x+1)^2 = x^2 + 2x + 1$

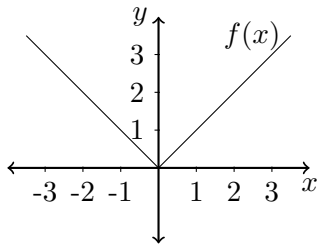
b) $h(x) = f(x-3)$ shifted right 3

$= (x-3)^2 = x^2 - 6x + 9$

c) $j(x) = f(x+\pi)$

$(x+\pi)^2 = x^2 + 2\pi x + \pi^2$ shifted left π

Problem 4. The graph of $f(x)$ appears below.



Match each of the following functions with the correct graph:

a) $f(x)+1$ iii

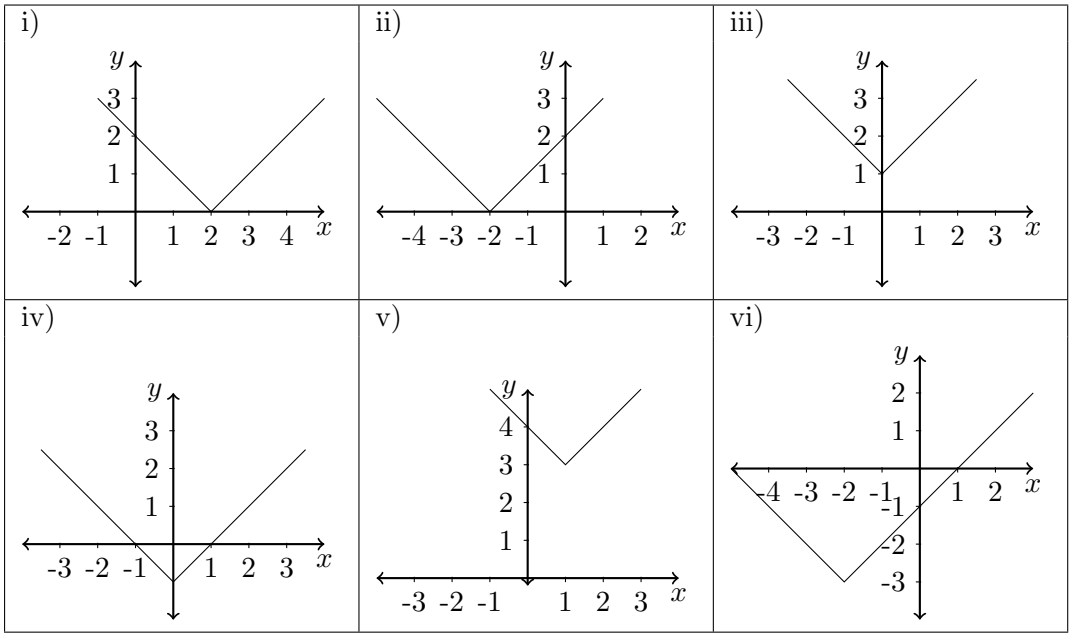
b) $f(x-1)+3$ ✓

c) $f(x-2)$ i

d) $f(x+2)-3$ vi

e) $f(x)-1$ iv

f) $f(x+2)$ ii



Problem 5. The cost of renting a limo has a flat fee of \$100 plus \$40 per hour.

a) Find a formula for $C(x)$, the total cost for renting a limo for x hours.

$C(x) = 100 + 40x$

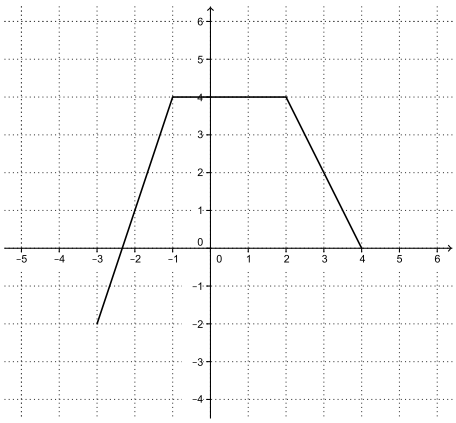
b) If the flat fee is raised by \$30, find a formula for the new total cost function, $D(x)$, in terms of $C(x)$.

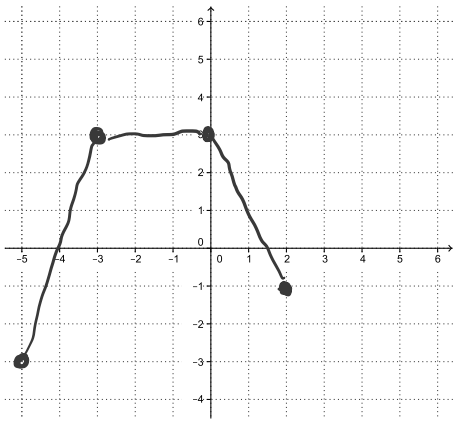
$D(x) = C(x) + 30$

c) The limo driver increases the flat fee by \$30, and leaves the cost per hour as \$40, but includes the first hour for free. Express $F(x)$, the new total cost, as a transformation of $C(x)$.

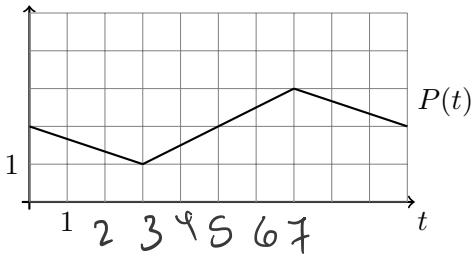
$F(x) = D(x-1) = C(x-1) + 30$

Problem 6. The function $g(x)$ is pictured below. Graph the function $g(x+2)-1$ on the provided set of axes.





Problem 7. Suppose that the population of Town A in thousands of people t years after 1990 is given by $P(t)$, shown in the graph below.



(a) Suppose Town B has a population given by $P(t)+3$. What is Town B's population in 1993? 1995? Interpret the population of Town B in terms of the population of Town A.

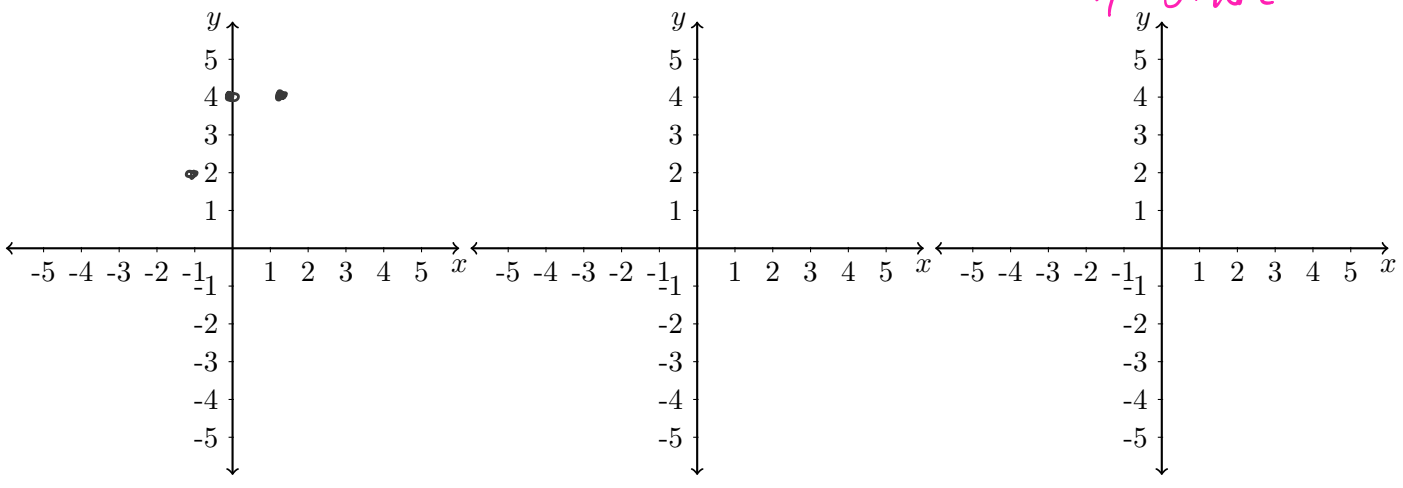
$P(3)+3 = 1+3 = 4$ $P(5)+3 = 2+3 = 5$

(b) Suppose Town C has a population given by $P(t-1)$. What is Town C's population in 1991? 1998? Interpret the population of Town C in terms of the population of Town A.

$P(1-1) = P(0) = 2$ $P(8-1) = P(7) = 3$

Problem 8. Let $f(x) = x^3 - x^2 + 4$. Graph each of the following functions. * Don't like that question

(a) $f(x)$ (b) $-f(x)$ (c) $f(-x)$



no graphing calculators allowed

Problem 9. Find a formula for the function whose graph is the graph of $f(x) = (x-1)^2$ reflected across the x-axis. Use your calculator to check your formula by graphing both it and the function $f(x)$ on the same axes.

$-f(x) = -(x-1)^2$

Problem 10. Find a formula for the function whose graph is the graph of $f(x) = (x-1)^2$ reflected across the y-axis. Use your calculator to check your formula by graphing both it and the function $f(x)$ on the same axes.

$f(-x) = (-x-1)^2 = (x+1)^2$

Problem 11. The graph of $y = f(x)$ contains the point $(1, 4)$. What point must lie on the resulting graph if the graph is

(a) reflected about the y-axis? $(-1, 4)$

(b) reflected about the x-axis? $(1, -4)$

(c) translated down 2 units? $(1, 2)$

(d) translated right by 1 unit? $(0, 4)$

(e) reflected about the x-axis, then shifted up by 1 unit? $(1,4) \to (1,-4) \to (1,-3)$

(f) translated up by 1 unit, then reflected about the x-axis? $(1,4) \to (1,5) \to (1,-5)$

Problem 12. Suppose $f(x)$ is an **even** function. Fill in as many missing values in the following table as possible. If it is not possible to fill in a value, explain why it is not.

$f(-x) = f(x)$

x	-6	-4	-2	0	2	4	6
$f(x)$	-1	2	5		5	2	-1

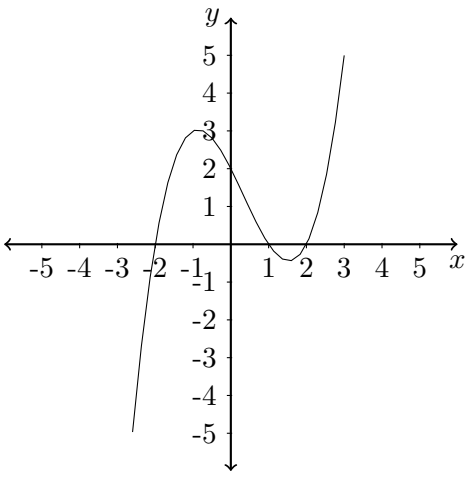
Problem 13. Suppose $f(x)$ is an **odd** function. Fill in as many missing values in the following table as possible. If it is not possible to fill in a value, explain why it is not.

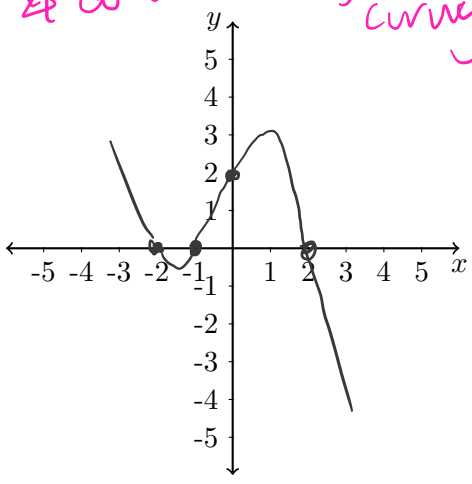
$f(-x) = -f(x)$

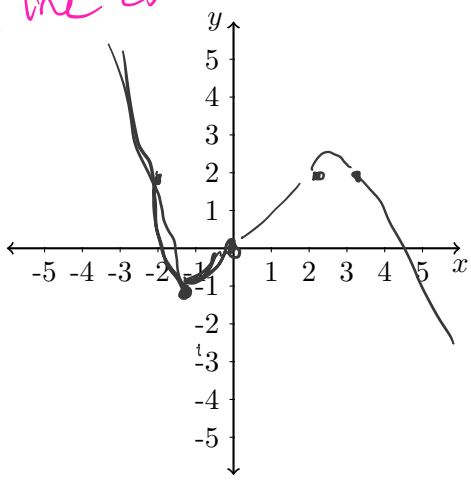
x	-6	-4	-2	0	2	4	6
$f(x)$	1	2	-5		5	-2	-1

Problem 14. The graph of $g(x)$ can be found below. On the first provided axes, graph $g(-x)$. On the second provided axes, graph $-g(x) + 2$.

do we really want a curvy line example?







Problem 15. Describe the shifts/reflections that can be applied to $f(x)$ in the correct order to obtain a graph of the following:

(a) $y = f(x-4)$

shift right 4

(b) $y = f(x+2)$

shift left 2

(c) $y = f(-x)$

reflect across y-axis

(d) $y = -f(-x)$

reflect y then reflect x

(e) $y = f(x+2) - 5$

left 2 then down 5

(f) $y = f(x) + 9$

up 9

Problem 16. Give an explicit formula for each of the transformations of $g(x) = x^2 + 2$ below. For example, $g(x) + 5 = x^2 + 2 + 5 = x^2 + 7$.

(a) $\frac{1}{4}g(x)$

$\frac{1}{4}x^2 + \frac{1}{2}$

(b) $-g(-x)$

$-x^2 - 2$

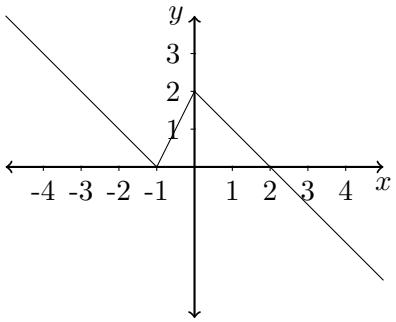
(c) $2g(x-5)$

$2(x-5)^2 + 4$

(d) $-\frac{3}{5}g(x+1)$

$-\frac{3}{5}(x+1)^2 - \frac{6}{5}$

Problem 17. The graph of $f(x)$ appears below.



Match each of the following functions with the correct graph:

a) $2f(x)$ iii

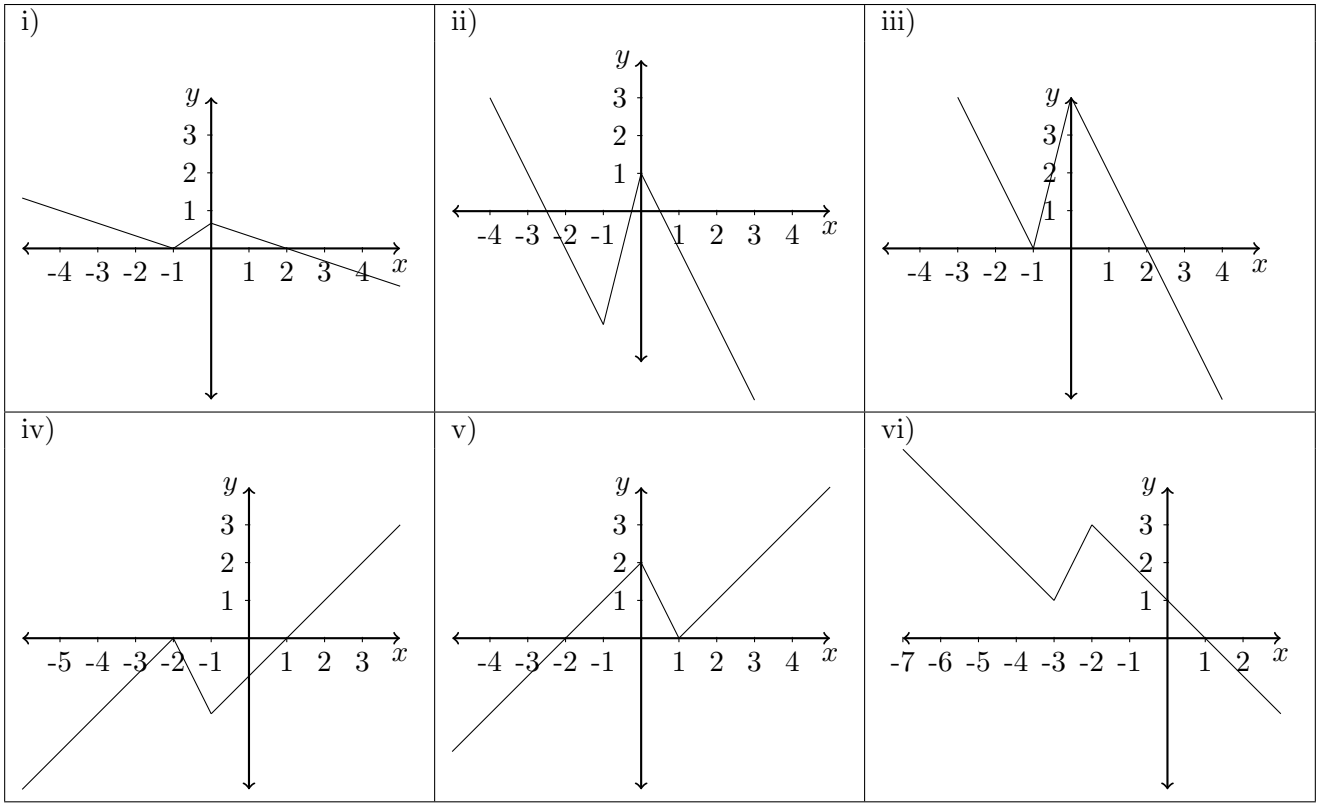
b) $\frac{1}{3}f(x)$ i

c) $f(-x)$ ✓

d) $f(x+2)+1$ ✓ i

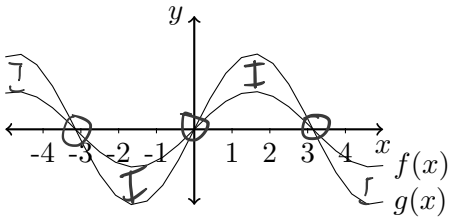
e) $-f(x+1)$ iv

f) $2f(x)-3$ ii



Problem 18. The following figure shows the graphs of $f(x)$ and $g(x)$. Is it possible for $f(x)$ to be a vertical compression of $g(x)$? Explain why or why not.

is it possible?



depends
are there
graphs the same?
are their intersections
the same?

Problem 19. Give an explicit formula for each of the transformations of $f(x) = x^2 + 2$ below. Then in each case determine the factor by which the graph of $f(x)$ is horizontally/vertically compressed/stretched. For example: $f(2x) = (2x)^2 + 2$ is a horizontal compression of $f(x)$ by a factor of 2.

(a) $6f(x)$ vert stret $\to 6$

$6(x^2+2)$

(b) $\frac{1}{3}f(3x)$ vert comp $\to 3$ hor comp 3

$\frac{1}{3}(9x^2+2)$

(c) $f(4x)$

$16x^2+2$ hor comp 4

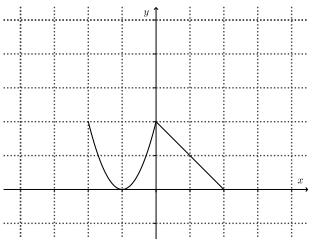
(d) $f\left(\frac{1}{2}x\right)$

$\frac{1}{4}x^2+2$ hor str 2

Problem 20. If the point $(3, 5)$ lies on the graph of $h(x)$, what point must lie on the graph of $h(3x)$? What point must lie on the graph of $3h(3x)$?

$h(3x)$: $(1, 5)$ $\quad$ $3h(3x)$: $(1, 15)$

Problem 21. The function $h(x)$ is pictured below. Match each of the following functions with the correct graph:



(a) $h(0.5x)$ vi

(b) $2h(x)$ ii

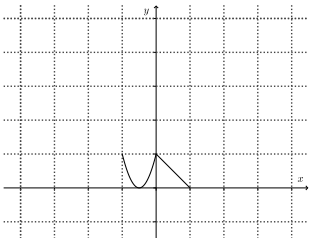
(c) $2h(0.5x)$ iii

(d) $0.5h(2x)$ i

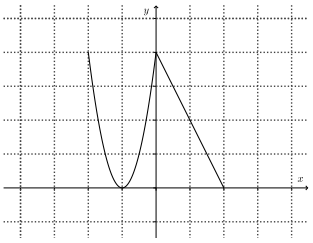
(e) $h(2x)$ iv

(f) $0.5h(x)$ ✓

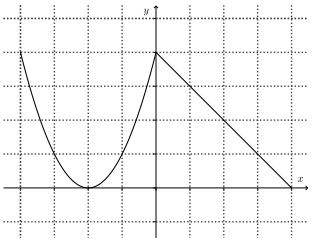
i)



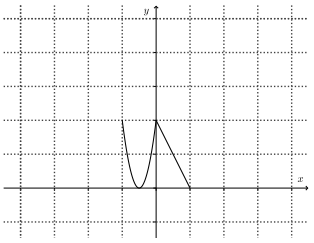
ii)



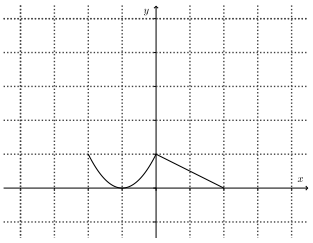
iii)



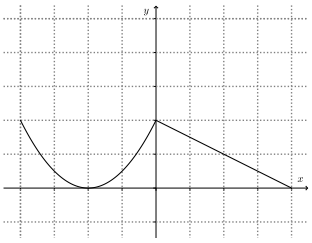
iv)



v)



vi)



Problem 22. Suppose the domain of a function $j(x)$ is $0 \leq x \leq 6$, and the range of $j(x)$ is $-3 \leq j(x) \leq 3$. Determine the domain and range of the following functions. *Hint: Use Problem 21 as an example.*

(a) $j\left(\frac{1}{4}x\right)$

D: $0 \leq x \leq 24$

R: $-3 \leq j(x) \leq 3$

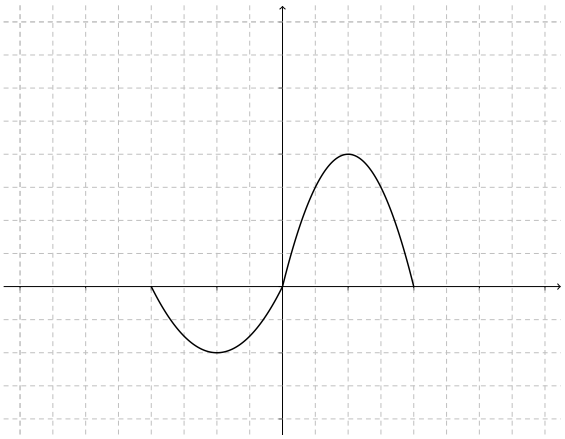
(b) $\frac{1}{3}j\left(2x\right)$

D: $0 \leq x \leq 3$

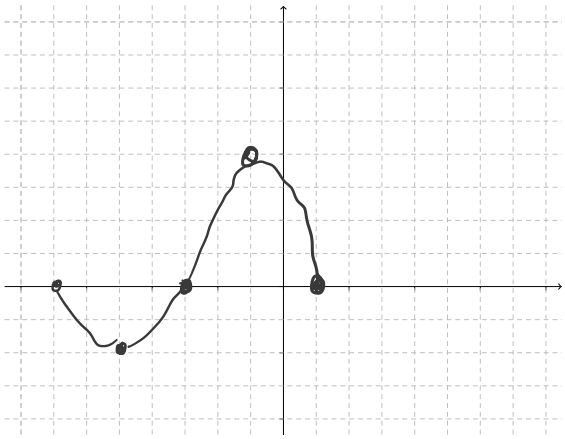
R: $-1 \leq j \leq 1$

Problem 23. The graph of $f(x)$ appears in the upper left square. Sketch the graph of the remaining functions.

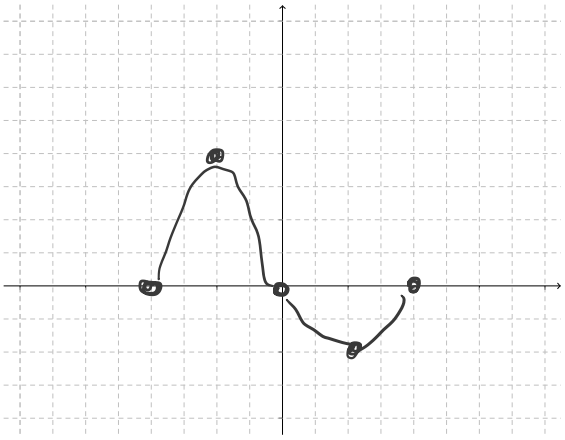
$f(x)$



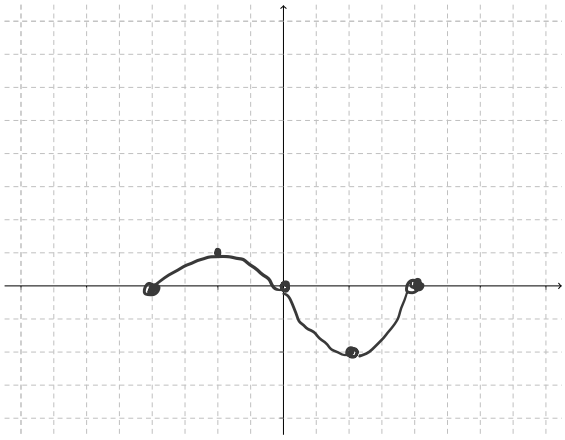
$f(x+3)-1$



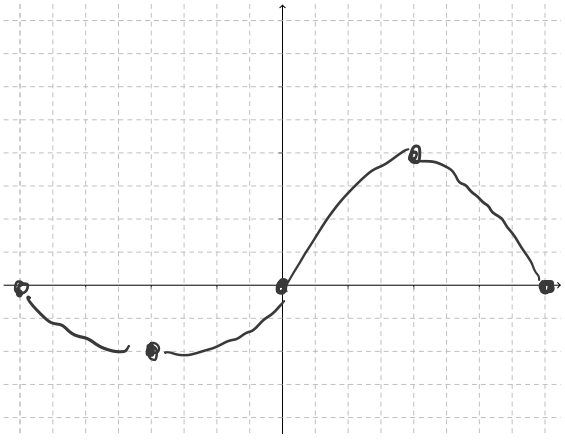
$f(-x)$



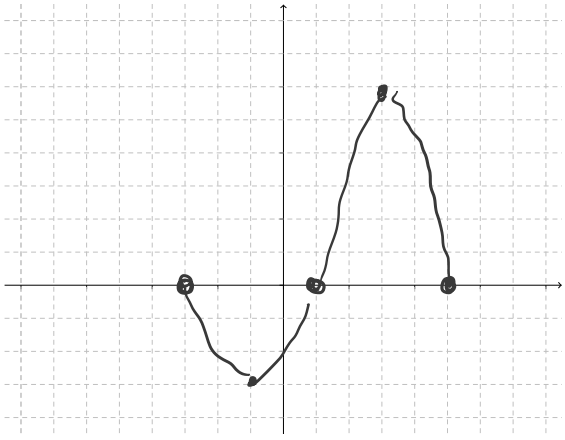
$-0.5f(x)$



$f(0.5x)$

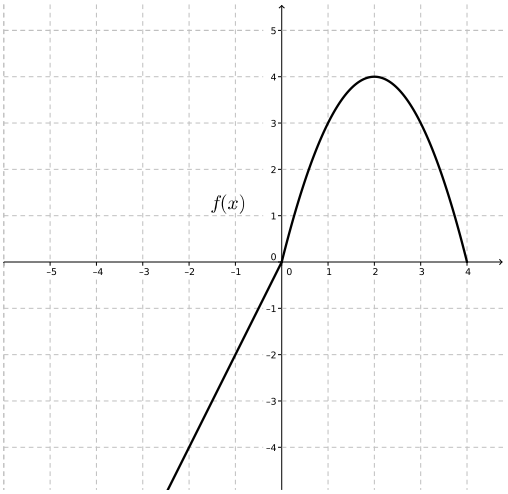


$1.5f(x-1)$

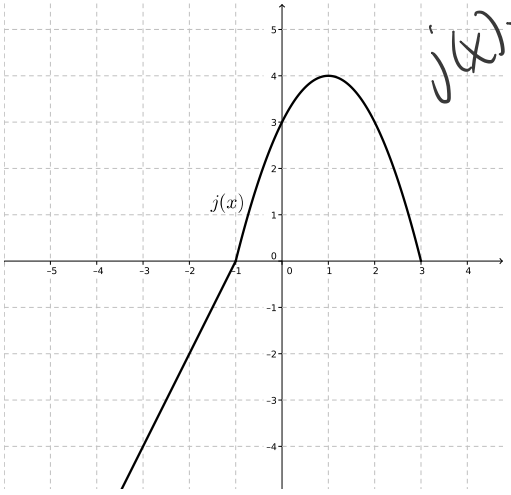


Additional Problems

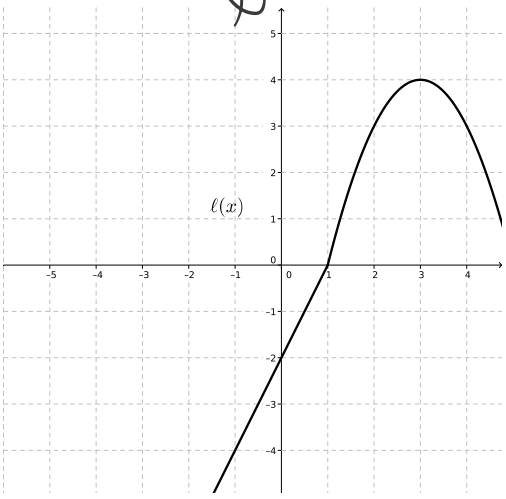
EP 1. The following is a graph of $f(x)$.



Find formulas for $j(x)$ and $\ell(x)$ in terms of $f(x)$.



$j(x) = f(x+1)$



$\ell(x) = f(x-1)$

EP 2. Sylvia is running a marathon. Let $D(t)$ represent the total distance in kilometers she has run t hours after starting. Find a formula for each of the following functions in terms of $D(t)$.

(a) $M(h)$, the total distance in **meters** she has run after h hours.

$M(h) = D(h) \cdot 1000$

(b) $K(m)$, the total distance in kilometers she has run after m **minutes**.

$K(m) = D\left(\frac{m}{60}\right)$

(c) $C(s)$, the total distance in **centimeters** she has run after s **seconds**.

$C(s) = M\left(\left(\frac{s}{120}\right)\right) \cdot 1000$